

DOCUMENT RESUME

ED 262 103

TM 850 591

AUTHOR Reckase, Mark D.
TITLE Models for Multidimensional Tests and Hierarchically Structured Training Materials. Final Report.
INSTITUTION American Coll. Testing Program, Iowa City, IA. Test Development Div.
SPONS AGENCY Office of Naval Research, Arlington, Va. Personnel and Training Research Programs Office.
REPORT NO ONR85-1
PUB DATE May 85
CONTRACT N00014-81-K0817
NOTE 33p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Instructional Development; *Item Analysis; *Latent Trait Theory; *Mathematical Models; Maximum Likelihood Statistics; Measurement Techniques; *Multitrait Multimethod Techniques; Psychometrics; Research Methodology; Testing; Test Items; Training
IDENTIFIERS *Learning Hierarchies

ABSTRACT

Work on item response theory was extended to two areas not extensively researched previously, including models for: (1) test items that require more than one ability for a correct response (MIRT); and (2) interaction between modules of instruction that have a hierarchical relationship (HST). In order to develop the MIRT and HST models, the author evaluated characteristics of the models, developed estimation procedures for the parameter of the models, and evaluated the models on their ability to describe real test data. A summary of the research is presented here, and references are made to papers and technical reports containing more detailed descriptions of the research efforts. (Author/LMO)

* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *

BEST COPY AVAILABLE

Final Report

Models for Multidimensional Tests and Hierarchically Structured Training Materials

Mark D. Reckase

Research Report ONR85-1
May 1985



The American College Testing Program
Assessment Programs Area
Test Development Division
Iowa City, Iowa 52243

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

- This document has been reproduced as received from the person or organization originating it.
- Minor changes have been made to improve reproduction quality.
- Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

Prepared under Contract No. N00014-81-K0817
with the Personnel and Training Research Programs
Psychological Sciences Division
Office of Naval Research

Approved for public release; distribution unlimited.
Reproduction in whole or in part is permitted for
any purpose of the United States Government.

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b RESTRICTIVE MARKINGS	
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release: distribution unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government	
2b DECLASSIFICATION/DOWNGRADING SCHEDULE		5 MONITORING ORGANIZATION REPORT NUMBER(S)	
4 PERFORMING ORGANIZATION REPORT NUMBER(S) ONR 85-1			
6a NAME OF PERFORMING ORGANIZATION ACT	6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION PERSONNEL & TRAINING RESEARCH PROGRAMS OFFICE OF NAVAL RESEARCH	
6c ADDRESS (City, State, and ZIP Code) P.O. Box 168 Iowa City, IA 52243		7b ADDRESS (City, State, and ZIP Code) Arlington, VA 22217	
8a NAME OF FUNDING/SPONSORING ORGANIZATION	8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-81-K0817	
8c ADDRESS (City, State, and ZIP Code)		10 SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO 61153N	PROJECT NO RR042-04
		TASK NO 042-04-01	WORK UNIT ACCESSION NO NR150-474
11 TITLE (Include Security Classification) Models for multidimensional tests and hierarchically structured training materials.			
12 PERSONAL AUTHOR(S) Mark D. Reckase			
13a TYPE OF REPORT Final Report	13b TIME COVERED FROM 81SEP01 TO 85FEB28	14 DATE OF REPORT (Year, Month, Day) 1985, May	15 PAGE COUNT 20
16 SUPPLEMENTARY NOTATION			
17 COSATI CODES		18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
		Item response theory Learning hierarchies Latent trait theory Multidimensional models	
19 ABSTRACT (Continue on reverse if necessary and identify by block number) Work on item response theory was extended to include two areas that had not been extensively researched previously. They include models for test items that require more than one ability for a correct response and models for the interaction between modules of instruction that have a hierarchical relationship. For both of these types of models, estimation procedures were developed for model parameters and extensive work was done to determine the appropriate interpretation of the parameter values. This report is a summary of work performed on these models over a three-year period.			
20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS		21 ABSTRACT SECURITY CLASSIFICATION	
22a NAME OF RESPONSIBLE INDIVIDUAL Dr. Charles Davis	22b TELEPHONE (Include Area Code) (202) 696-4046	22c OFFICE SYMBOL	

Contents

	Page
Introduction.....	1
Development and Evaluation of MIRT Models.....	2
Analysis of the General Rasch Model.....	6
Interpretation of the Model Parameters.....	11
Summary and Conclusions.....	13
Models for Performance on Hierarchically Structured	
Training Materials.....	14
The Module Characteristic Curve Model.....	16
Summary and Conclusions.....	18
References.....	20

Final Report
Models for Multidimensional Tests
and Hierarchically Structural Training Materials

Since the 1950's, there has been increasing interest in psychological and educational measurement that is based upon probabilistic models of the interaction between a person and a test item. These model-based procedures demonstrate how strong assumptions can be used to gain increased control over the measurement process. For example, using item response theory (IRT), the precision of measurement at every point along an ability scale can be determined. Also, items can be selected from a pool to form a test with any desired level of precision at any point on the score scale.

The strong assumptions needed for these model-based procedures are basically that the probabilistic model that has been selected accurately reflects the test data, and that local independence holds for the model. This latter assumption means that the response to one item does not affect the response to another item, and that the response by one person does not affect the response by another person.

Most of the current models assume that the measuring instrument measures only a single trait (Rasch, 1960; Lord, 1952; Birnbaum, 1968). For many tests, this assumption is at least approximated, and for other tests, it is unlikely to be met at all. Most of the current models also are limited to describing a person's response to a single item. In some cases this limitation may make it difficult to solve some measurement problems.

The purpose of the research done on this contract was to extend the types of models available for model-based measurement. Two types of extensions were considered. The first was an extension of item response theory models to the

case where the measurement device was not assumed to be measuring a single dimension. These models were labelled multidimensional item response theory (MIRT) models.

The second type of extension was to cases where sets of related items were considered as a unit. These related sets of items were assumed to be measuring educational constructs that could be arranged into a hierarchy that facilitated learning. These models could be used to determine the interrelationship between the constructs in the hierarchy and the level that must be reached on each construct before a person should be moved on to the next higher level of the hierarchy. Models for tests used with hierarchically arranged instructional units were labelled models for hierarchically structured tests (HST).

The approach taken to develop and evaluate the MIRT and HST models was to first logically evaluate the characteristics of potential models, then to develop estimation procedures for the parameter of the models, and finally to evaluate the models on their ability to describe real test data. These steps were performed separately for a wide class of models of each type. The results of the research will now be described for each type of model, with the analysis of the MIRT models being presented first. Only a summary of the outcome of the research will be presented here, but references will be made to papers and technical reports that contain the details of the research efforts.

The Development and Evaluation of MIRT Models

The class of possible multidimensional, probabilistic models of the interaction between a person and a test item is essentially infinite in

size. Any expression that maps a vector of abilities into a probability could be considered as a MIRT model.

Therefore, the first step in the research effort was to limit the possible models to a manageable subset. This was done by reviewing the literature to determine what MIRT models had been proposed. The review identified three general classes of models that had been suggested for use with multidimensional data.

The first of the classes of models considered were extensions of the general model proposed by Rasch (1961). This model, in its most general form, is given by

$$P(x_{1j} | \theta_j, \sigma_1) = \frac{1}{\gamma(\theta_j, \sigma_1)} e^{[\phi(x_{1j})' \theta_j + \psi(x_{1j})' \sigma_1 + \theta_j' \chi(x_{1j}) \sigma_1 + \rho(x_{1j})]} \quad (1)$$

where $P(x_{1j} | \theta_j, \sigma_1)$ is the probability of response x_{1j} given the values of vector parameters θ_j and σ_1 ; θ_j is a vector of parameters that describes the characteristics of person j ; σ_1 is a vector of parameters that describes item i ; $\gamma(\theta_j, \sigma_1)$ is a normalizing function defined by

$$\gamma(\theta_j, \sigma_1) = \sum_{x_{1j}} e^{[\phi(x_{1j})' \theta_j + \psi(x_{1j})' \sigma_1 + \theta_j' \chi(x_{1j}) \sigma_1 + \rho(x_{1j})]} \quad (2)$$

that ensures that the sum of the probabilities of the responses to this item is equal to 1.0; $\phi(x_{1j})$ is a vector of scoring weights that indicates the value to be given to each response to the items when considering the estimation of the ability parameters; $\psi(x_{1j})$ is a vector of scoring weights that indicates the value to be given to each response to the item when considering the estimation of item parameters; $\chi(x_{1j})$ is a matrix of scoring weights that indicates the value to be given to different products of the

elements of δ_j and σ_1 ; and $\rho(x_{1j})$ is a constant that is used to set the origin of the linear function defined by the exponent. This equation defines a very general class of models that specifies the dimensionality of the complete latent space by a linear function in the exponent of the logistic model form. Note that this model allows one ability to compensate for another in the metric of θ_j . That is, a high value of θ_{j1} can compensate for a low value of θ_{jn} in the linear function of θ_j defined by

$$\psi_1(x_{1j})\theta_{j1} + \psi_2(x_{1j})\theta_{j2} + \dots + \psi_m(x_{1j})\theta_{jm} \quad (3)$$

The same type of linear compensation is present for the item parameters.

The second class of models considered was proposed by Mulaik (1972). This class of models is of the form

$$P(x_{1j} | \theta_j, \sigma_1) = \frac{\sum_{k=1}^m e^{(\theta_{jk} + \sigma_{1k})x_{1j}}}{1 + \sum_{k=1}^m e^{(\theta_{jk} + \sigma_{1k})}} \quad (4)$$

where $x_{1j} = 0, 1$; m is the number of dimensions; and all of the other terms have been defined previously. This model specifies the dimensionality of the complete latent space as a sum of exponential terms. Ability and item parameters can also compensate for each other in this model, but the compensation occurs on an exponential scale. An interesting point to note is that if each exponent is zero in this model, the probability of a correct response is $m/(m + 1)$. Thus, as the number of dimensions, m , increases, the

probability of a correct response increases unless all of the person and item parameters are rescaled. For the model presented in Equation 1, the probability is always .5 when the exponent is zero.

The third class of models that was considered was proposed by Symson (1978) and in a slightly different form by Whitely (1980). This class of models is of the general form given by

$$P(x_{ij}=1|\theta_j, a_i, b_i, c_i) = c_i + (1-c_i) \prod_{k=1}^m \frac{e^{a_{ik}(\theta_{jk} - b_{ik})}}{1 + e^{a_{ik}(\theta_{jk} - b_{ik})}} \quad (5)$$

where a_i is a vector of discrimination parameters, b_i is a vector of difficulty parameters, c_i is the lower asymptote of the probability function, and all of the other terms have been defined previously. This class of models determines the probability of a response based on abilities in a multidimensional space as the product of a series of probability like terms. These terms are, in effect, the probability of the response to the item if the item only required the one dimension. The overall probability is the product of the probabilities on each dimension. If the exponent is zero on each dimension, the probability will be $c_i + (1 - c_i) (.5)^m$. Thus, the probability of a correct response will be reduced as each additional dimension is included, unless the parameters are rescaled for each level of dimensionality.

Since the models given in Equations 4 and 5 both require a rescaling of the ability scales with each change in dimensionality, and because both of these models present some very difficult problems in parameter estimation, they were removed from initial consideration and the model presented in Equation 1 became the focus of research effort.

Analysis of the General Rasch Model

The model presented in Equation 1 defines a very rich class of special cases. By selectively setting the weight functions to zero, many different possible models can be derived, each of which have different properties. Each of these special cases was studied both through a mathematical analysis of the equation for each model and through a statistical analysis of simulated data generated using each model. The results of these analyses were reported in a technical report and in a series of papers presented at professional meetings. The full references to the report and the papers are given below.

McKinley, R. L. and Reckase, M. D. (1982). The use of the general Rasch model with multidimensional item response data (Research Report ONR 82-1). Iowa City, IA: The American College Testing Program.

McKinley, R. L. and Reckase, M. D. (1982, March). Multidimensional latent trait models. Paper presented at the meeting of the National Council on Measurement in Education, New York.

McKinley, R. L. and Reckase, M. D. (1982, May). An analysis of the characteristics of a family of IRT models. Paper presented at the meeting of the Psychometric Society, Montreal.

The results of these analyses showed that two special cases of the general Rasch were capable of modeling realistic multidimensional item response data. The first case uses only the $\theta_j' x_{1j} \sigma_1$ and $\psi(x_{1j})' \sigma_1$ terms

of the general model. The weights for the other terms were set to zero. The model for this case is given by

$$P(x_{ij} | \theta_j, \sigma_1) = \frac{1}{\gamma(\theta_j, \sigma_1)} e^{(\sum_{k=1}^m \sigma_{ik} \theta_{jk} + \sum_{k=1}^m \sigma_{i,m+k})} \quad (6)$$

where the symbols have been defined earlier. This form of the model can be written in the more familiar form given by

$$P(x_{ij} | \theta_j, a_1, d_1) = \frac{e^{(\sum_{k=1}^m a_{ik} \theta_{jk} + d_1)}}{1 + e^{(\sum_{k=1}^m a_{ik} \theta_{jk} + d_1)}} \quad (7)$$

where $a_{ik} = \sigma_{ik}$, $d_1 = -\sum_{k=1}^m a_{ik} b_{ik} = \sum_{k=1}^m \sigma_{i, m+k}$, $1 + e^{(\sum_{k=1}^m a_{ik} \theta_{jk} + d_1)} = \gamma(\theta_1, \sigma_1)$ and a_{ik} and b_{ik} can be interpreted as the a- and b-parameters from unidimensional IRT models. Equation 7 can also be thought of as a multidimensional extension of the two-parameter logistic model; therefore, it has been labelled the M2PL model.

The second special case of the general Rasch model that was found to model multidimensional item response data uses only the $\phi(x_{ij})' \theta_j$ and $\psi(x_{ij})' \sigma_1$ terms from the general model. This model is of the form

$$P(x_{ij} | \theta_j, \sigma_1) = \frac{1}{\gamma(\theta_j, \sigma_1)} e^{(\phi(x_{ij})' \theta_j + \psi(x_{ij})' \sigma_1)} \quad (8)$$

where all of the terms have been defined previously. This model has been labelled the "cluster model" because in order for it to model multidimensional data, x_{ij} must be the response string for a cluster of items rather than the response to a single item. If the item cluster contains two dichotomously scored items, the possible x_{ij} responses would be 0,0; 0,1; 1,0; and 1,1. For each of these responses, a different weight function would be available for the θ - and σ -vectors.

Although the cluster model was very promising, it had one difficulty that made it less attractive. In order to use the model, items had to be clustered, and no rigorous means for doing the clustering has been developed. Therefore, research efforts concentrated on the M2PL model.

Estimation of Model Parameters

In order for a model to be useful, it must be possible to estimate the parameters of the model. Once the M2PL model was selected as the model for further research efforts, work was begun on developing procedures for estimating the model parameters. Two different approaches were taken to solve the estimation problem: (a) unconditional maximum likelihood, and (b) conditional maximum likelihood. Once computer programs were developed for these two approaches, they were validated using both simulated test data generated from the M2PL model, and real test data that were selected because of their multivariate properties. The estimation procedures and the results of the program validation studies were presented in the publications and papers listed below.

McKinley, R. L. and Reckase, M. D. (1983). MAXLOG: a computer program for the estimation of the parameters of a multidimensional logistic model.

Behavior Research Methods and Instrumentation, 15(3), 389-390.

McKinley, R. L. and Reckase, M. D. (1983). An application of a multidimensional extension of the two-parameter logistic latent trait model (Research Report ONR83-3). Iowa City, IA: The American College Testing Program.

Reckase, M. D. and McKinley, R. L. (1982, July). Some latent trait theory in a multidimensional latent space. Paper presented at the Invitational Conference on IRT/CAT, Wayzata, MN.

Reckase, M. D. and McKinley, R. L. (1982, August). The feasibility of a multidimensional latent trait model. Paper presented at the meeting of the American Psychological Association, Washington, D.C.

McKinley, R. L. (1983, April). A multidimensional extension of the two-parameter logistic latent trait model. Paper presented at the meeting of the National Council on Measurement in Education, Montreal.

McKinley, R. L. and Reckase, M. D. (1983, April). The use of IRT analysis on dichotomous data from multidimensional tests. Paper presented at the meeting of the American Educational Research Association, Montreal.

The results of these studies showed that both the unconditional and conditional maximum likelihood procedures could be used to estimate the item and ability parameters of the M2PL model, but that the unconditional maximum likelihood procedure required somewhat less computer time. However, both procedures require fairly extensive computer facilities, and as the number of dimensions in the model increased, the computer time required became prohibitive. It was clear that improved estimation procedures were needed if the M2PL model was to be widely used.

The validation of the estimation procedures yielded uniformly good results when simulated test results were used. However, when real test data were analyzed, the results were inconsistent. Some studies gave readily interpretable results that were in many ways similar to factor analytic results. In other studies anomalies appeared, such as highly negatively correlated ability estimates that suggested that added constraints were needed to control the estimation process.

In order to study the estimation process in more detail, the M2PL procedure was used to analyze simulated test data that had been produced using a multivariate ability distribution that had varying degrees of correlation between the abilities. The results of the study were presented in the following report.

McKinley, R. L. and Reckase, M. D. (1984). An investigation of the effect of correlated abilities on observed test characteristics (Research Report ONR 84-1). Iowa City, IA: The American College Testing Program.

The study showed that the dimensionality of both the items and the examinee population was important in interpreting the results of an M2PL analysis. If each item were a relatively pure measure of an ability, the procedure obtained good estimates of the ability parameters, even when they were correlated. But, as the correlation between ability estimates increased, there was some deterioration of the accuracy of the estimates. When each item measured more than one ability, the effect of correlated abilities was more extreme. As the correlation between abilities increased, the M2PL solution tended to collapse to a single dimension. The results seemed to imply the need for procedures for oblique rotations to improve the recovery of the ability dimensions.

Interpretation of the Model Parameters

When a MIRT model is used, estimates can be obtained for the ability and the item parameters. The ability parameter estimates can be interpreted in a fairly straightforward manner as the amount of ability a person has on each dimension. The item parameter estimates, however, do not have the same intuitive meaning. Therefore, a major part of this project dealt with determining the MIRT model analogs to the unidimensional IRT item parameters and the measures of quality, such as item and test information. The results of the work in this area were presented in the following documents.

McKinley, R. L. and Reckase, M. D. (1983). An extension of the two-parameter logistic model to the multidimensional latent space (Research Report ONR83-2). Iowa City, IA: The American College Testing Program.

Reckase, M. D. and McKinley, R. L. (1983, April). The definition of difficulty and discrimination for multidimensional item response theory models. Paper presented at the meeting of the American Educational Research Association, Montreal.

Reckase, M. D. and McKinley, R. L. (1983, June). The item difficulty concept generalized to the multidimensional latent space. Paper presented at the meeting of the Psychometric Society, Los Angeles.

Reckase, M. D. and McKinley, R. L. (1984, June). Multidimensional difficulty as a direction and a distance. Paper presented at the meeting of the Psychometric Society, Santa Barbara, CA.

Initial work in this area concentrated on deriving a direct generalization of the interpretations of the difficulty and discrimination parameters and item and test information from the unidimensional item response theory models to the MIRT models. Since the difficulty of an item was defined for the unidimensional models as the point on the ability scale corresponding to the point of inflection of the item characteristic curve, multidimensional difficulty was conceptually thought of as the point of inflection of the multidimensional item response surface (IRS). An analysis of this approach quickly made two important points evident. First, for an IRT there is not a single point of inflection, but rather a locus of points of inflection. Depending upon the MIRT model and the dimensionality being considered, this locus of points of inflection could be a straight line, a curve, a hyperplane, or a hypersurface. The complexity of the locus of points of inflection made its practical application difficult.

The second point that became evident was that the locus of points of inflection could change with the direction taken relative to the surface in the multidimensional space. This is a direct consequence of the fact that the slope at a point on the IRS is different in different directions. The direction in the space is one way of indicating the composite of abilities that is of interest.

In order to take these two points into account, a definition of multidimensional difficulty was derived that was based upon a vector conceptualization. The multidimensional difficulty of an item was defined as the direction from the origin of the multidimensional space to the point of steepest slope and the distance from the origin to the point of steepest slope. Discrimination of an item was related to the slope in the difficulty direction at the point of the steepest slope. Information was also given a directional interpretation. For a group centered at the origin of the space, an item is most informative in the difficulty direction. The item information can also be determined in any other direction, but the maximum information will be less than in the direction indicated by the multidimensional difficulty.

The definitions of multidimensional difficulty, discrimination, and information are general enough that they apply to any MIRT model that is monotonically increasing in probability with an increase in any ability dimension. The definition also includes the unidimensional definitions as special cases.

Summary and Conclusions

This portion of the research project accomplished several important tasks in the development of MIRT. A number of models were analyzed and the

multidimensional extension of the two-parameter logistic model was selected as a promising model for future work. Estimation procedures were developed for this model and the results were validated using simulated and real test data. A theoretical foundation was laid for an interpretation of the item parameters of the MIRT models, and definitions of multidimensional item difficulty, discrimination, and information were developed. At this point, a sufficient framework has been developed to make multidimensional item response theory a viable technique.

Although substantial advances have been made in the area of MIRT, even more work is left to be done. The current estimation programs require excessive amounts of computer time when more than two or three dimensions are specified for a model. Work needs to be done to make estimation of the parameter more efficient. Procedures are needed to determine the appropriate number of dimensions for a set of test data, and procedures for indicating the fit of the models to the data are needed. A related question is whether the M2PL model is an accurate representation of the interaction between a person and an item. This model implies that one ability can compensate for another. Perhaps a model of this type is not appropriate. These and other questions will be addressed in future work.

Models for Performance on Hierarchically Structured Training Materials

Programs of instruction are often composed of many short, homogenous instructional units that have been arranged according to the logical interrelationships of the content. In many cases, short tests are given to determine each student's level of competence on a unit of instruction, and the

scores on the tests are used to route the students through the units of instruction. The purpose of this component of the project was to evaluate an IRT-type model that had potential for assisting in determining the interrelationships between the instructional units and in determining the decision points that should be used with each unit test to minimize routing error. The model treats each unit, or module, of instruction as a complex item and hypothesizes a particular mathematical form for the interrelationship between performance on one module and the probability of successfully passing the next module in the instructional program.

The first step in the evaluation of this model for performance in instructional programs was to review the literature in the area called "learning hierarchies" to determine what procedures were currently being used to evaluate the interrelationships between units of instruction and to set passing scores on the unit tests. The information obtained from the review would serve as a basis for comparison for the results obtained from the proposed model. The review of the literature was presented in the following report.

Reckase, M. D. and McKinley, R. L. (1982). The validation of learning hierarchies (Research Report ONR 82-2). Iowa City, IA: The American College Testing Program.

The review of the literature indicated that there were two general types of procedures that had been used to indicate the relationships between instructional units; those based on coefficients of dependence, and those based on a more complete description of the relationships between units of instruction, usually a mathematical model. The procedures based on

coefficients of dependence were found to provide insufficient information for validating the sequence of instructional units, or for setting passing scores. The procedures based on mathematical models were found to have more potential, but the currently available procedures did not seem to meet the needs of instructional programs. There seemed to be a clear need for a procedure that could be used to arrange units of instruction into a hierarchy based upon the prerequisite knowledge required by each unit of instruction, and that could be used to set passing scores for each unit that would improve the efficiency and accuracy of the routing process. The model proposed and evaluated during this research effort was designed to perform these functions.

The Module Characteristic Curve Model

The basic idea behind the proposed model for the interrelationship between modules of instruction is that if two modules form a learning hierarchy, performance on the higher level instructional module is dependent upon prerequisite knowledge obtained from the lower level module of instruction. Thus, if sufficient knowledge has not been gained on the lower level module, a high level of performance cannot be exhibited on the higher level module of instruction. This implies that success on the higher module is related to the level of performance on the lower module.

The probabilistic model that was hypothesized to describe the relationship between hierarchically related instructional modules is given by

$$P_j(\theta_{1k}) = c_j + (1 - c_j - e_j) \frac{e^{D_{aj}(\theta_{1k} - b_j)}}{1 + e^{D_{aj}(\theta_{1k} - b_j)}} \quad (9)$$

where $P_j(\theta_{ik})$ is the probability of passing module j given level of performance θ_{ik} of examinee i on prerequisite module k , c_j is the probability of passing module j if the examinee has not acquired any knowledge in module k , e_j is the probability of passing module j if the examinee has mastered module k , $D = 1.7$, a_j is a parameter related to the strength of the relationship between the two modules, and b_j is the difficulty of the passing score used on module j . This model predicts the probability that an examinee will pass module j based on his/her performance on module k .

In order to use this model, estimates of achievement are first obtained on module k . This can either be done by analyzing the module k test using an IRT model, or by converting the raw scores on module k to z -scores. These achievement measures are then used as known values and the model parameters are estimated using a maximum likelihood estimation procedure.

A very low a -parameter estimate is an indication that the two modules are not very highly related. A high a -value indicates that knowledge on module k is very important for module j . A high estimate for the c -parameter indicates that examinees can perform well on module j even without mastering module k . A low c -value indicates that an examinee cannot perform well on module j unless knowledge has been acquired on module k .

Estimates of the e -parameter indicate the maximum probability of passing the j module given that the examinee has mastered module k . Low values indicate that module k contains only a small portion of the information needed to pass module j . High values indicate that module k includes most of the information needed to pass module j .

The b -parameter estimates indicate the point on the module k scale that best distinguishes between persons who pass or fail module j . This point will change with changes in the passing score on module j . The point on the module

k scale specified by the b-parameter is the suggested decision point on module k for routing to module j if misclassification errors in either direction are considered equally serious.

In order to evaluate this model, it was applied to both simulated and real test data to determine whether the estimation procedures worked properly, and whether it realistically represented actual test results. The outcome of these studies were presented in the following documents.

McKinley, R. L. and Reckase, M. D. (1984). A latent trait model for sequentially arranged units of instruction. Iowa City, IA: The American College Testing Program.

McKinley, R. L. and Reckase, M. D. (1984, April). A latent trait model for use with sequentially arranged units of instruction. Paper presented at the meeting of the American Educational Research Association, New Orleans.

The studies showed that the parameters of the model could be accurately estimated and that for one set of real test data, the model gave very reasonable results. There was some indications, however, that the upper and lower asymptote parameters might not be needed. It may be possible to simplify the model to a two-parameter logistic form.

Summary and Conclusions

A model for the relationship between modules of instruction that are hierarchically related was proposed and evaluated using both simulated and real test data. The results of the studies showed that the model parameters could be accurately estimated and that the model was a good representation of

real test data that should be hierarchically related. However, the upper and lower asymptotes did not appear to be needed for the particular real data set that was analyzed. Further studies need to be done to determine whether this is a general finding applicable to all hierarchically arranged modules, or whether it only applies to this case. If the c- and e-parameters are not needed, the model can be simplified to a two-parameter logistic model.

One problem with the use of the model became evident with the analysis of the real test data. In order to accurately estimate the parameters of the model, examinees must be routed to the higher level unit of instruction even when they have not performed well on the lower level unit. This is poor educational practice and, in many cases, this data collection procedure cannot be followed. This makes it difficult to obtain data for use in estimating the parameters of the model. It may be that the model will have to be modified to accommodate the routing procedures that are currently being used in modularized instructional programs.

References

- Birnbaum, A. (1968). Some latent trait models and their use in inferring an examinee's ability. In F.M. Lord and M.R. Novick, Statistical theories of mental test scores. Reading, MA: Addison-Wesley.
- Lord, F.M. (1952). A theory of test scores. Psychometric Monograph, 7.
- Mulaik, S.A. (1972, March). A mathematical investigation of some multidimensional Rasch models for psychological tests. Paper presented at the meeting of the Psychometric Society, Princeton, NJ.
- Rasch, G. (1960). Probabilistic models for some intelligence and attainment tests. Copenhagen: Danish Institute for Educational Research.
- Rasch, G. (1962). On general laws and the meaning of measurement in psychology. Proceedings of the Fourth Berkely Symposium on Mathematical Statistics and Probability, 4, 321-334.
- Sympson, J.B. (1978). A model for testing with multidimensional items. In D.J. Weiss (Ed.), Proceedings for the 1977 Computerized Adaptive Testing Conference. Minneapolis: University of Minnesota.
- Whitely, S.E. (1980). Measuring aptitude processes with multicomponent latent trait models (Technical Report No. NIE-80-5). Lawrence, KS: University of Kansas, Department of Psychology.

Distribution List

Personnel Analysis Division
AF/MPXA
5C360, The Pentagon
Washington, DC 20330

Air Force Human Resources Lab
AFHRL/MPD
Brooks AFB, TX 78235

Air Force Office
of Scientific Research
Life Sciences Directorate
Bolling Air Force Base
Washington, DC 20332

Dr. Robert Ahlers
Code N711
Human Factors Laboratory
NAVTRAEQUIPCEN
Orlando, FL 32813

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Technical Director
Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

Special Assistant for Projects
OASN(M&RA)
5D800, The Pentagon
Washington, DC 20350

Dr. Alan Baddeley
Medical Research Council
Applied Psychology Unit
15 Chaucer Road
Cambridge CB2 2EF
ENGLAND

Dr. Patricia Baggett
University of Colorado
Department of Psychology
Boulder, CO 80309

Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

CDR Robert J. Biersner, USN
Naval Biodynamics Laboratory
P. O. Box 29407
New Orleans, LA 70189

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
Israel

Dr. Werner Birke
Personalstammamt
der Bundeswehr
D-5000 Koeln 90
WEST GERMANY

Code N711
Attn: Arthur S. Blaiwes
Naval Training Equipment Center
Orlando, FL 32813

Dr. R. Darrell Bock
University of Chicago
Department of Education
Chicago, IL 60637

Dr. Nick Bond
Office of Naval Research
Liaison Office, Far East
APO San Francisco, CA 96503

Dr. Robert Breaux
Code N-095R
NAVTRAEQUIPCEN
Orlando, FL 32813

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. Patricia A. Butler
NIE Mail Stop 1806
1200 19th St., NW
Washington, DC 20208

Dr. James Carlson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

Dr. Robert Carroll
NAVOP 01B7
Washington, DC 20370

Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

Director
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Scientific Advisor
to the DCNO (MPT)
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Chief of Naval Education
and Training
Liason Office
AFHRL
Operations Training Division
Williams AFB, AZ 85224

Assistant Chief of Staff
Research, Development,
Test, and Evaluation
Naval Education and
Training Command (N-5)
NAS Pensacola, FL 32508

Office of the Chief
of Naval Operations
Research Development
& Studies Branch
NAVOP 01B7
Washington, DC 20350

Dr. Stanley Collyer
Office of Naval Technology
800 N. Quincy Street
Arlington, VA 22217

Dr. Hans Crombag
University of Leyden
Education Research Center
Boerhaavelaan 2
2334 EN Leyden
The NETHERLANDS

CTB/McGraw-Hill Library
2500 Garden Road
Monterey, CA 93940

CDR Mike Curran
Office of Naval Research
800 N. Quincy St.
Code 270
Arlington, VA 22217-5000

Mr. Timothy Davey
University of Illinois
Educational Psychology
Urbana, IL 61801

Dr. Dattprasad Divgi
Syracuse University
Department of Psychology
Syracuse, NY 13210

Dr. Hei-Ki Dong
Ball Foundation
800 Roosevelt Road
Building C, Suite 206
Glen Ellyn, IL 60137

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Susan Embertson
University of Kansas
Psychology Department
Lawrence, KS 66045

ERIC Facility-Acquisitions
4833 Rugby Avenue
Bethesda, MD 20014

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. Pat Federico
Code P13
NPRDC
San Diego, CA 92152

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52240

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Bob Frey
Commandant (G-P-1/2)
USCG HQ
Washington, DC 20593

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01002

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134

Dipl. Pad. Michael W. Habon
Universität Dusseldorf
Erziehungswissenschaftliches
Universitätsstr. 1
D-4000 Dusseldorf 1
WEST GERMANY

Dr. Ron Hambleton
School of Education
University of Massachusetts
Amherst, MA 01002

Prof. Lutz F. Hornke
Universität Dusseldorf
Erziehungswissenschaftliches
Universitätsstr. 1
Dusseldorf 1
WEST GERMANY

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

Mr. Dick Hoshaw
NAVOP-135
Arlington Annex
Room 2834
Washington, DC 20350

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

Dr. Earl Hunt
Department of Psychology
University of Washington
Seattle, WA 98105

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Douglas H. Jones
Advanced Statistical
Technologies Corporation
10 Trafalgar Court
Lawrenceville, NJ 08148

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. Norman J. Kerr
Chief of Naval Education
and Training
Code 00A2
Naval Air Station
Pensacola, FL 32508

Dr. William Koch
University of Texas-Austin
Measurement and Evaluation
Center
Austin, TX 78703

Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Patrick Kyllonen
AFHRL/MOE
Brooks AFB, TX 78235

Dr. Anita Lancaster
Accession Policy
OASD/MI&L/MP&FM/AP
Pentagon
Washington, DC 20301

Dr. Daryll Lang
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Jerry Lehnus
OASD (M&RA)
Washington, DC 20301

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Alan M. Lesgold
Learning R&D Center
University of Pittsburgh
Pittsburgh, PA 15260

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat 23
9712GC Groningen
The NETHERLANDS

Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

Dr. Robert Lockman
Center for Naval Analysis
200 North Beauregard St.
Alexandria, VA 22311

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. James Lumsden
Department of Psychology
University of Western Australia
Nedlands W.A. 6009
AUSTRALIA

Dr. William L. Maloy (02)
Chief of Naval Education
and Training
Naval Air Station
Pensacola, FL 32508

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Clessen Martin
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Dr. Scott Maxwell
Department of Psychology
University of Notre Dame
Notre Dame, IN 46556

Dr. Samuel T. Mayo
Loyola University of Chicago
820 North Michigan Avenue
Chicago, IL 60611

Dr. James McBride
Psychological Corporation
c/o Harcourt, Brace,
Javanovich Inc.
1250 West 6th Street
San Diego, CA 92101

Dr. Clarence McCormick
HQ. MEPCOM
MEPCT-P
2500 Green Bay Road
North Chicago, IL 60064

Dr. Barbara Means
Human Resources
Research Organization
1100 South Washington
Alexandria, VA 22314

Dr. Robert Mitlevy
Educational Testing Service
Princeton, NJ 08541

Dr William Montague
NPRDC Code 13
San Diego, CA 92152

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152

Headquarters, Marine Corps
Code MPI-20
Washington, DC 20380

Director
Research & Analysis Division
Navy Recruiting Command (Code 22)
4015 Wilson Blvd.
Arlington, VA 22203

Program Manager for Manpower,
Personnel, and Training
NAVMAT 0722
Arlington, VA 22217-5000

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

Dr. William E. Nordbrock
FMC-ADCO Box 25
APO, NY 09710

Dr. Melvin R. Novick
356 Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Director, Manpower and Personnel
Laboratory
NPRDC (Code 06)
San Diego, CA 92152

Library
Code P201L
Navy Personnel R&D Center
San Diego, CA 92152

Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harry F. O'Neil, Jr.
Training Research Lab
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

Mathematics Group
Office of Naval Research
Code 744MA
800 North Quincy Street
Arlington, VA 22217-5000

Office of Naval Research
Code 442PT
800 N. Quincy Street
Arlington, VA 22217-5000
(5 Copies)

Special Assistant for Marine
Corps Matters
Code 100M
Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217-5000

Commanding Officer
Army Research Institute
ATTN: PERI-BR (Dr. J. Orasanu)
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
AFHRL/MOAN
Brooks AFB, TX 78235

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dr. Roger Pennell
Air Force Human Resources
Laboratory
Lowry AFB, CO 80230

Administrative Sciences Department
Naval Postgraduate School
Monterey, CA 93940

Department of Operations Research
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Mr. Robert Ross
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Lawrence Rudner
403 Elm Avenue
Takoma Park, MD 20012

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
Knoxville, TN 37916

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152

Dr. Robert Sasmor
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152

Dr. W. Steve Sellman
OASD(MRA&L)
2B269 The Pentagon
Washington, DC 20301

Dr. Sylvia A. S. Shafto
National Institute of Education
1200 19th Street
Mail Stop 1806
Washington, DC 20208

Dr. Joyce Shields
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

Dr. Richard Snow
Liaison Scientist
Office of Naval Research
Branch Office, London
Box 39
FPO New York, NY 09510

Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

Dr. William Stout
University of Illinois
Department of Mathematics
Urbana, IL 61801

Maj. Bill Strickland
AF/MPXOA
4E168 Pentagon
Washington, DC 20330

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152

Dr. John Tangney
AFOSR/NL
Bolling AFB, DC 20332

Dr. Kikumi Tatsuoka
CERL
252 Engineering Research
Laboratory
Urbana, IL 61801

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
Department of Statistics
University of Missouri
Columbia, MO 65201

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 310
St. Paul, MN 55114

Dr. Frank Vicino
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Howard Wainer
Division of Psychological Studies
Educational Testing Service
Princeton, NJ 08540

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Mr. Thomas A. Warm
Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Dr. Brian Waters
HumRRO
300 North Washington
Alexandria, VA 22314

Dr. Edward Wegman
Office of Naval Research
Code 411
800 North Quincy Street
Arlington, VA 22217-5000

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

Dr. Donald Weitzman
MITRE
1820 Dolley Madison Blvd.
MacLean, VA 22102

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 12
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Rand R. Wilcox
University of Southern
California
Department of Psychology
Los Angeles, CA 90007

German Military Representative
ATTN: Wolfgang Wildegrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
Army Research Institute
5001 Eisenhower Ave.
Alexandria, VA 22333

Ms. Marilyn Wingersky
Educational Testing Service
Princeton, NJ 08541

Dr. Martin F. Wiskoff
Navy Personnel R & D Center
San Diego, CA 92152

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Major Frank Yohannan, USMC
Headquarters, Marine Corps
(Code MPI-20)
Washington, DC 20380

Dr. Joseph L. Young
Memory & Cognitive
Processes
National Science Foundation
Washington, DC 20550